

The process of thinking by prospective teachers of mathematics in making arguments

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ABSTRACT

This study aimed to describe the process of thinking by prospective teachers of mathematics in making arguments. It was a qualitative research involving the mathematics students of STKIP PGRI Jombang as the subject of the study. Test and task-based semi structural interview were conducted for data collection. The result showed that 163 of 260 mathematics students argued using inductive and deductive warrants. The process of thinking by the prospective teachers of mathematics in making arguments had begun since they constructed their very first idea by figuring out some objects to make a conclusion. However, they also found a rebuttal from that conclusion, though they did not further describe what such rebuttal was. Therefore, they decided to construct the second ideas in order to verify the first ones through some pieces of definition.

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1. INTRODUCTION

Thinking is a mental activity that individuals experience while solving problems [1-6]. After interpreting the problem well, the next step to do by the problem-solver is selecting the best strategy as solution. They (i.e., the problem solver) implement the selected stages to find the best solution for their problems.

Solving problems started from constructing, clarifying, and evaluating ideas, respectively [7]. When the problem-solvers solve a problem, they are trying to construct ideas as solutions for the problem by thinking some alternatives of problem solving and figuring out those constructed ideas. Such ideas should be explained by considering their similarities and differences, combining the similar ones, and separating the distinctive ones. However, some of those ideas are inconsistent with the definition, theorem, and postulates, and thus, they should be evaluated in order to make the best decision for solving the problem.

Education should provide chances for students to develop various skills and competences including the competence of thinking, particularly the capability of having argument [8, 9]. In other word, education should not only focus on the process of students' thinking, but also their capability of making arguments. In addition to learning the meaning of a concept, students should learn to choose among difference options and explain the reason behind their choices.

Students make arguments to justify the solutions and actions they made in solving problems [10-16]. Argumentation, however, is a capability to connect particular data for a claim purpose [8]. It is an important part of informal reasoning as the center of intellectual competences for solving problems, making judgment and decision, as well as constructing ideas and assurance [12]. Furthermore, the competence of making arguments is an essential component to improve the performance of problem-solving [17]. Following those several perspectives, a problem-solver needs to have argumentations to decide, achieve, and bolster a

reasonable solution for problem solving. Improving the competence of making arguments is necessary for students to make them capable to describe some reasons of whether strengthening or refusing a perspective, view, or idea. Having such competence, students may leave any hesitancy and vacillation behind, and thus, solve the problem. They may also be free to choose and propose a reasonable solution.

The process of an individual making arguments should be analyzed using a more fruitful format so that he/she might not solely distinguish the premises and conclusions [18]. Therefore Toulmin proposed a layout of argument known as Toulmin's Scheme. It consists of Data (D), claim (C), Warrant (W), Backing (B), Rebuttal (R), and Qualifier (Q). Data refers to some facts to support the claim proposed. Claim is a proposition with supporting data. Data and Claim derive from a condition in which "if D, then C". Warrant is an assurance for data to support the claim. It has Backing as its supporter. Backing provides further legal-based evidence as the basis of warrant. Rebuttal is an exceptional condition for arguments, and Qualifier may reveal the strength level of the data proposed for the claim by warrant. Toulmin's Scheme to analyze arguments is presented in Figure 1.

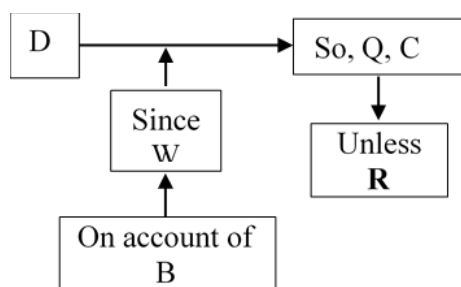


Figure 1. Toulmin's scheme for analyzing arguments

Warrant types include inductive, structural-intuitive, and deductive [19, 20] and so on. Inductive warrant is a base derived from the process of evaluating one or more specific cases. Structural-intuitive warrant is a base derived from the result of intuition (i.e., intuitive thinking) on the structure of an individual's internal representation. Deductive warrant is a base derived from the process of formal mathematics justification in order to assure the general conclusion made. Again, deductive warrant is a formal mathematics justification to assure a general conclusion [19]. The justification is through several ways, such as axial cuts, algebraic manipulation, or the examples of denying. Deductive was a thought derived from general things implemented and led to more specific ones [21]. Deductive was a process of thinking derived from an existing proposition led to the new one in the form of conclusion [22]. With such deductive thinking, individuals may start their departure from a theory, principle or conclusion they think it is right and general in nature. at student-level, it is expected to use deductive warrant in the process of logically explaining, justifying, and reasoning their arguments in order to ensure, strengthen, or refuse other arguments, perspectives, or ideas. Therefore, this study focused on the process of thinking of prospective teachers of mathematics in making arguments with deductive warrant initiated by inductive one.

The utilization of Toulmin's Scheme for analyzing arguments in education of mathematics was seen in [23] study which formulated the profile of students' arguments to solve mathematical problems. That profile is helpful for teachers to have better understanding on students' approaches in solving problems. [24] described the concept of evidence by prospective teachers to justify their arguments in class. Another study using Toulmin's Scheme was [25] that evaluated the position of rebuttal. It shows that the previous studies tended to focus on individuals' arguments in class discussion (i.e., argument in dialogue) rather than argument not-in-dialogue. Thus, this present study would specifically discuss the second one; argument not-in-dialogue. It is important as individuals try to reveal and ensure the right perspective using arguments led to themselves. They try to ensure themselves. With this competence (i.e., argument not-in-dialogue), they may get ready for any argument in-dialogue.

The students of mathematics education program were selected as the subject of this study, as they would be the prospective teachers of mathematics and might contribute to the development of students' thinking in mathematical argumentation. Students' arguments depended on the construction of theorem habit in class, the characteristics of tasks, and the kinds of particular reasoning by teachers [26]. Teachers' encouragement for students to explain, note, and justify their arguments in a class discussion aimed to improve students' capability on argumentation [27]. The concept of evidence by the teachers of mathematics in junior high schools may affect their teaching practices in assisting students to make arguments [24].

Following the previous theory and studies on the process of thinking and mathematical argumentation, this present study aimed to describe the process of students' thinking in making arguments. It would be analyzed using the framework of Toulmin's Scheme, that each of the components within would be further analyzed with Swatz's theory.

2. RESEARCH METHOD

The sixth-semester students of mathematics education program in STKIP PGRI Jombang participated as the subject of this study. They are the prospective teachers of mathematics for junior and high school levels. They were selected as they had learned the concept of relation, indicating that they were capable to solve the given problems. The selected students were those making arguments with deductive and inductive warrants.

First, the students were given a mathematical problem to be solved by optimally expressing their thought during the process of problem solving (i.e, think aloud). Second, a task-based semi-structural interview was conducted. In broadly speaking, this interview aimed to see what kind of thought the subject was thinking of while inferring something and taking a step. The question might be in the form of "How do you think of this?" or "what are you thinking of right now?" The questions also aimed to see their reason in taking some steps to solve the given problem.

The data collection was through test and semi-cultural interview. Thus, the instrument of this study was a task of mathematics and interview manual. The task was aimed to describe the subject's arguments based on the kinds of warrant, as seen in Figure 2. It was adapted from [20]. Some information along with statements the subject needed to investigate was provided within the instrument. Furthermore, they were asked to express their thought aloud during the process of completing the given task. Video recording was conducted to see the subjects' activities during the process of problem-solving.

Dedefinition 1. Given that S is a partially ordered set of binary relation R on S , and A is the subset of S .
 A is called *chain* if each of two distinctive elements; a, b ; on A meets one of either $a R b$ or $b R a$.

Definition 2. S is a partially ordered set of binary relation R on S , and B is the subset of S . B is called *antichain*, if each of two distinctive elements; a, b ; on B meets bot $a \not R b$ and $b \not R a$.

Investigate the truth of the statement "If P is not *chain*, then P is *antichain*"!

Figure 2. Mathematical task

3. RESULTS AND ANALYSIS

260 undergraduate students of mathematics education program of STKIP PGRI Jombang participated in this study. The kinds of warrant they used were presented in Table 1.

Table 1. Categorization of Students' Warrant-Based Arguments

Kind of Warrant	The Quantity of Student
Inductive	56
Structural-intuitive	15
Deductive	5
Structural inductive and deductive	21
Inductive and deductive	163

Table 1 showed that 62.69% of the prospective teachers of mathematics made their arguments using inductive and deductive warrant. The following described the process of their thinking in making arguments with inductive and deductive warrant.

3.1. Constructing the First Idea

The process of thinking by the subject in making arguments to investigate the truth of a given statement had already started since they mentioned the information provided, including: " P is not chain". That information was data for making conclusion that " P is antichain". The following showed the subject's *think aloud*.

The process of thinking by prospective teachers of mathematics in making arguments (Lia Budi Trisanti)

Subject: "P is not chain, then P does not meet the condition in which 'if each of two distinctive elements; a, b; is on A, then P would be qualified'. However, P does not meet the condition either a R b or b R a. may it be called antichain?"

The subject's constructed idea to investigate the mathematical statement was by making the examples of partially ordered set S and identifying those examples using the definition of *chain* and *antichain*. They had such idea from the previous experience with identic task.

The process of explaining the idea by the subject started from providing the example of set $S = \{1, 2, 3, 4, 5, 6, 7\}$ and binary relation $R = \{(a, b) | a \text{ is the factor of } b, a, b \in S\}$. Subsequently, they made and identified the subset of S using the definition of *chain* and *antichain*, respectively. The identification aimed to show that the example successfully met one of two possibilities: (i) if P is not *chain*, then P is *antichain*; or (ii) if P is *chain*, then P is *antichain*. Thus, the subject's argument used inductive warrant. Figure 3 presented the subject's written answer showing argument with inductive warrant.

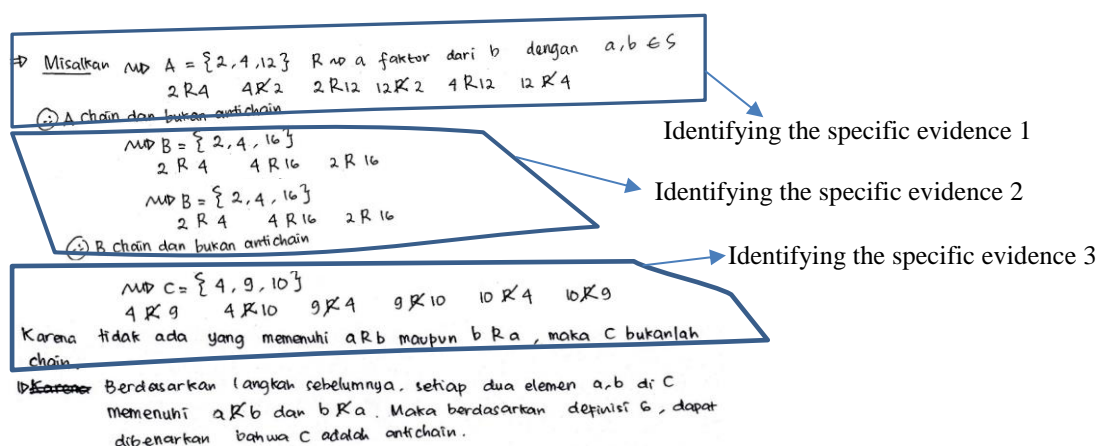


Figure 3. The subject's written result

Following [28] perspective, the process of thinking by the subject was on basic level, in which they used their logical reasoning through multiplication, division, or addition. Following the way of the subject's experiment was classified into *naïve empiricism*, in which they asserted the truth of their result in making conclusion after verifying some cases [29]. However, that argument was classified into informal argument as the warrant was based on the concrete interpretation of mathematical concepts [30]. The concrete representation by group 1 was making specific examples.

3.2. Evaluating the Fairness of the First Idea

At the stage of evaluating the fairness of their idea, the subject expressed the legal basis (i.e., backing) of their idea. They claimed that the backing was in the form of a subset not classified into chain, and thus indeed classified into antichain, as well as the vice versa.

The subject concluded that if P was not chain, then P was antichain. The level of trust (i.e., qualifier) for that conclusion was still probable, in which if P was most likely not chain, then P would be antichain. It was because the subject still saw S as a set consisting of 7 members. Another possibility was that, for instance, S was a set of complex and real numbers.

The conformity between qualifier and the kind of warrant in the subject's argument was consistent with [20] that inductive warrant aimed to alleviate the uncertainty of conclusion. The following presented the subject's statement on an interview.

Qualifier
Rebuttal

Subject : I thought that there might be an exception as the conclusion remained likely true, Mam.

The subject's exploration showed that there was likely a rebuttal on the conclusion. However, they did not specifically describe what such rebuttal was.

3.3. Constructing the Second Idea

The second idea constructed by the subject was to investigate the truth of formally mathematical statement. The process of explaining this idea started since they negated the definition of chain. The negation of the definition of *chain* was the definition of “*non-chain*”. The subject compared definitions between “*non-chain*” and “*antichain*”. They realized that the definition of “*non-chain*” is not similar to the definition of “*antichain*”. Hence, they argued that if P was not *chain*, it was still in the air that P was *antichain*. Therefore, they made some examples of denying for the statement “if P was not *chain*, then P was *antichain*”. The examples of denying by the subject was set $A = \{2, 3, 4\}$ and himpunan $B = \{4, 9, 16\}$ with binary relation $R = \{(a, b) | a \text{ the factor of } b, a, b \in S\}$. Thus, the argument used deductive warrant. They concluded that the statement “of P was not *chain*, then P was *antichain*” was wrong. Figure 4 presented the subject’s written result showing the argument with deductive warrant.

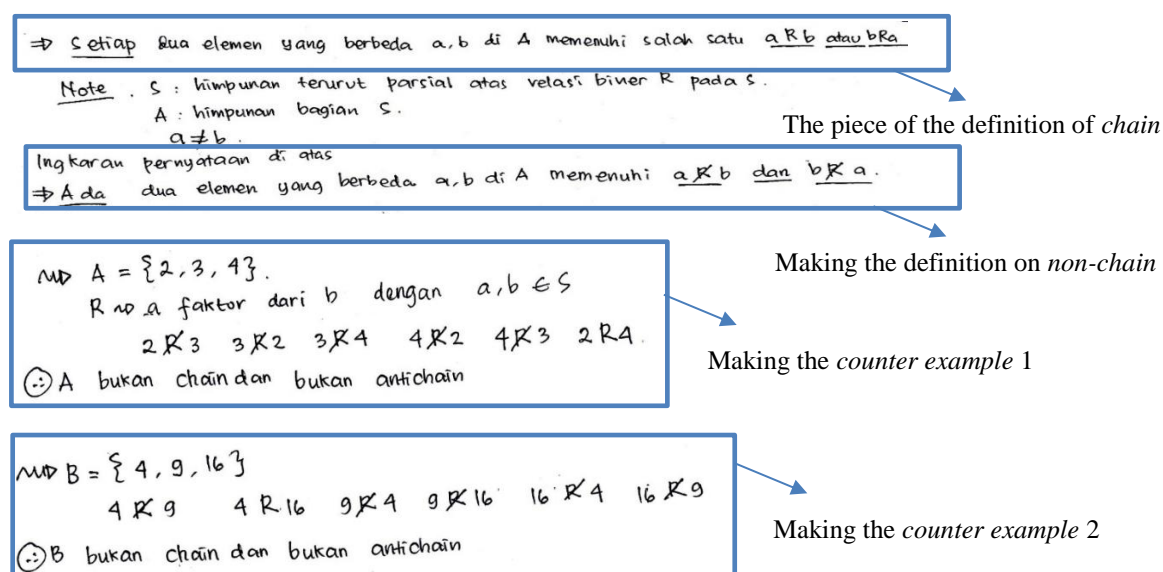


Figure 4. The subject’s written result

That argument was classified into formal one as the warrant was based on the definition, axiom, and theorem [30]. The subject used the formal argument to eliminate any hesitancy, and thus ensure the truth of the statement.

3.4. Clarifying the Ideas

At this stage, the subject did not combine those constructed ideas to ensure the conclusion in case of comparing the definition of *non-chain* and *antichain* as well as the *counterexample*. The reason not combining those ideas was because they revealed different conclusions. Thus, the subject put aside those which had different conclusions. Nevertheless, the ideas were used as the conclusion’s rebuttal. It was consistent with [31] that *inductive* frame was constructed at the initial phase of deductive frame. In addition, *deductive* frame might be useful to either support or rebut inductive warrant.

3.5. Evaluating the Fairness of the Ideas

The subject re-read their written result and adjusted each of their ideas with the given information. Those ideas were justified based on the applied rule. The basic rule (i.e., *backing*) was in the form of negation of both *quantor* and plural statements. The subject inferred that if P was not *chain*, then it was still in the air that P was *antichain*. They defined that the statement was wrong as not all *non-chain* were *antichain*. They found a *rebuttal* on their ideas, in which the conclusion would not be applicable if each of the elements of A was not interrelated. The subject revealed the *qualifier* from their conclusion in the form of *certain*. The conformity between the *qualifier* and the kinds of *warrant* in the argument did exist. It was consistent with [20] that deductive warrant was used to alleviate any hesitancy and uncertainty on the conclusion. The following showed the result of the subject’s think aloud. Table 2 interpreted the thinking path of such argument, as follow.

Subject : If P is not *chain*, then P is *antichain*, and thus, the inconformity disappears, oh wait... rather than disappeared, it is used as a rebuttal in which each of the elements of P is not interrelated, and thus, it would meet the statement 'if P is not *chain*, then P is *antichain*. Thus, it is clear that the statement 'if P is not *chain*, then P is *antichain* is wrong.

Rebuttal

Conclusion

Qualifier

Table 2. The Stage of Thinking and the Scheme of the Subject's Argument

The Stage of Thinking	The Scheme of Mathematical Argumentation	Explanation
Constructing the first idea		The subject defined the <i>data</i> (D) and <i>claim</i> (C) based on the given information on task, and then identified some specific cases to make a general conclusion. Therefore, they used inductive warrant (W_i) to ensure the data for the claim.
Evaluating the fairness of the first idea		The subject revealed the <i>backing</i> (B_i) ensuring the truth of the first idea. The qualifier (Q_i) from the conclusion was <i>probable</i> . They found an exceptional condition (<i>rebuttal</i>) (R_i) in that conclusion. Nevertheless, they did not specifically describe what such rebuttal was.
Constructing the second idea		Constructing the second idea was aimed to strengthen the conclusion of the first idea. It was to verify the conclusion derived from a theory or principle that they considered true, and thus, they used deductive warrant (W_d) on their argument.
Clarifying the first and the second ideas		At this stage, the subject combined between inductive (W_i) and deductive (W_d) warrants to ensure the data for the claim (C). Those two warrants, however, might reveal different conclusions, and thus, it made deductive warrant act against the conclusion derived from inductive warrant.
Evaluating the fairness of the ideas		The subject revealed the <i>backing</i> (B_d) to ensure the truth of deductive warrant (W_d). In addition to <i>backing</i> , the subject also revealed a <i>rebuttal</i> (R_d), indicating that no exceptional condition existed on the conclusion. The <i>Qualifier</i> (Q_d) of the conclusion derived from deductive warrant was <i>certain</i> . Thus, the qualifier and conclusion derived from deductive warrant may against the qualifier and conclusion derived from inductive warrant.

4. CONCLUSION

The process of thinking by undergraduate students in making arguments started from the stage of constructing the first idea, in which they expressed the components of arguments, including Data, Claim, and Warrant. The idea was by taking some objects in order to make conclusion. At the stage of evaluating the first idea, they showed the components of Qualifier, Backing, and Rebuttal. The level of trust (i.e., qualifier) of the conclusion derived from inductive warrant was *probable*. Furthermore, they found an exceptional condition (i.e., rebuttal) on the conclusion. However, they did not specifically describe what the rebuttal was. At the stage of constructing the second idea, the students verified and formulated a conclusion using

deductive warrant. Their current argument with this second idea was derived from a theory, principle, and/or conclusion they considered true or general in nature. The justification was from some axial cuts, algebraic manipulation, and counterexample. At the stage of clarifying the ideas, the students connected inductive warrant to deductive warrant. These two warrants were connected when they had similar conclusion. Inductive warrant would be put aside when the conclusion revealed was different from deductive warrant. At the stage of evaluating the fairness of the second idea, the students showed the components of Backing, Qualifier, and Rebuttal. They eliminated any possible rebuttal as the qualifier of the conclusion was certain. Thus, deductive warrant might improve the assurance of the conclusion revealed; from being *probable* to *certain*. Overall, deductive warrant might either strengthen or weaken the conclusion derived from inductive warrant.

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